Influence of internal tides on sea level variations at the Suruga Bay coast*

Masaji Matsuyama** and Atsushi Ohwaki**

**Abstract**: The influence of internal tides on the sea level elevation was studied by using the sea level and subsurface temperature records observed at Uchiura, at the head of Suruga Bay in summer 1978. The amplitudes of sea surface elevations due to the internal tides (SLA) are defined as difference between the observed and predicted sea levels. A comparison of the time series of the temporal variations of amplitude and phase for the tidal periods of SLA with those of the subsurface temperatures shows that the characteristics of SLA are similar to those of the subsurface temperature due to the internal tides. The maximum amplitude among the four major tidal constituents of SLA is estimated to be 4.0-5.3 cm for the $M_2$ constituent and the minimum is 0.1-1.0 cm for the $K_1$ constituent. The sea level records at four stations, Uchiura, Shimizu, Minamizu and Omaezaki, are analyzed by the same method as well. The $M_2$ constituent of SLA is the largest among the four major tidal constituents at Uchiura and Shimizu, near the head of Suruga Bay during this period. The $M_2$ constituent is the largest at Uchiura, and the maximum value of the $M_2$ amplitude at Uchiura is about twice as high as that at Shimizu, about 2.5 times that at Minamizu, and about 4 times the value at Omaezaki. The $M_2$ constituent of SLA at Shimizu is almost out of phase with that at Uchiura. This relation of the amplitude and phase between Uchiura and Shimizu is supported by a numerical experiment of the internal tides in Suruga Bay. The amplification of the $M_2$ constituent agrees with the numerical experiments and is considered to be due to resonance with the longitudinal internal seiche of Uchiura Bay. These results suggest that the SLA variations with tidal periods, especially the $M_2$ constituent, are mostly induced by the internal waves.

1. Introduction

Sea surface elevations are well known to be influenced by internal waves, but their amplitude due to long internal waves (i.e. the internal tides and seiches) is on the order 10 of the amplitude of thermocline displacement from a rough estimation (e.g. Proudman, 1954; Phillips, 1977). Therefore, these magnitudes are really a few centimeters at most and are much less than those due to the surface waves. Therefore, this influence has been neglected for the analysis of sea level variations in many cases. But, in coastal areas where large amplitude internal tides exist frequently, the influence is considered to prevent precise estimation of the surface elevation due to the surface tides.

In Suruga Bay, which is located at the central Japan island and faces the Pacific Ocean (Fig. 1), it is feared that a huge earthquake will occur in the near future, so that an observation system has been established to receive any precursory indications of an earthquake. Sea level observations are being made at some sites along the bay coast part of this effort. Matsuyama and Teramoto (1985) and Matsuyama (1985 a) reported that internal tides in Uchiura Bay (Fig. 1) have amplitudes of thermocline vertical displacement larger than ten meters near the head of the bay at times in summer and early fall. In Uchiura Bay, the amplitudes of the surface tide for the four major constituents ($M_2$, $S_2$, $K_1$, $O_1$) range from 15 to 41 cm at the coast (Table 1). Therefore, the sea surface variations due to the internal tides can possibly reach 1/10 of those due to the surface tides in summer and early fall. Inaba (1982) suggested the presence of internal tides at other regions in Suruga Bay from the current measurements. On the other hand, Tamura et al. (1986) analyzed the sea-level records at Omaezaki during the period from 1970 to 1979 and showed that the $M_2$ constituent has seasonal variations with maximum value in

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Table 1. Harmonic constants of the four major constituents (Tide Table by Japan Oceanographic Data Center).

<table>
<thead>
<tr>
<th></th>
<th>M_2</th>
<th>S_2</th>
<th>K_1</th>
<th>O_1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>P</td>
<td>A</td>
<td>P</td>
</tr>
<tr>
<td>Uchiura</td>
<td>41.3</td>
<td>167</td>
<td>18.9</td>
<td>192</td>
</tr>
<tr>
<td>Shimizu</td>
<td>40.2</td>
<td>166</td>
<td>18.6</td>
<td>192</td>
</tr>
<tr>
<td>Minamizu</td>
<td>39.6</td>
<td>163</td>
<td>18.2</td>
<td>188</td>
</tr>
<tr>
<td>Omaezaki</td>
<td>41.9</td>
<td>166</td>
<td>19.0</td>
<td>192</td>
</tr>
</tbody>
</table>

A: amplitude (cm). P: phase (degree).

summer and minimum in winter. The amplitudes of the seasonal variations reach about 4 per cent of the M_2 amplitude. These variations agree with those of the internal tides, so that it is speculated to be due to the internal tides.

Investigations of the influence of internal tides on sea surface elevations are required, because surface elevations due to the internal tides are unpleasant noise in the analysis of sea level records at the coast. This study tries to show, through the analysis of the hourly sea level data, that internal tides in Suruga Bay have an influence on sea level variations at the bay coast. First, we define the sea-level anomaly (SLA) as the difference between the predicted and observed sea levels. We describe the time and spatial variations of SLA with the semidiurnal and diurnal periods during the period from 1 July to 2 September 1978. Long-term temperature measurements for investigating the characteristics of internal tides were made in the subsurface layer at the head of Uchiura Bay from 14 July to 22 August 1978 (Matsuyama, 1985a). Second, the time series of SLA will be compared with those of temperature to examine whether the sea-level anomaly (SLA) can be used for the analysis of internal tides. The spatial variations of the semidiurnal constituent of SLA are also compared with results obtained from numerical experiments of internal tides in Suruga Bay (Matsuyama, 1985b).
2. Data and data analysis

Hourly sea level records are taken at Uchiura (Sta.U), Shimizu (Sta.S), Omaezaki (Sta. OM), and Minamiizu (Sta.M). Uchiura and Shimizu are located near the head of Suruga Bay, and Minamiizu and Omaezaki near the mouth of the bay (Fig. 1). The harmonic constants, i.e. the amplitude and phase of tidal constituents, for the sea level at each station prepared by JODC (Japan Oceanographic Data Center) are calculated from the records collected for 10 years from 1971 to 1980 (Table 1). We define the hourly sea-level anomaly (SLA) by subtracting the predicted sea level (PSL) from the observed hourly sea level (RSL) in the following form:

\[ RSL - PSL = SLA \]

PSL are constructed from 39 tidal constituents. The time series of hourly SLA can be obtained at each tidal station. We will focus on the diurnal and semidiurnal constituents of the SLA variations. These variations are expected to be mainly related to the internal tides, and variations of the atmospheric pressure and wind. These atmospheric phenomena are supposed to have an influence mainly on the sea level with the diurnal period, but it is not easy to remove them from the SLA variations because it is difficult to estimate the response time of the sea level against the various atmospheric systems. Therefore, in this study, we do not neglect the effects of wind and atmospheric pressure on the sea level variations, as a first step.

3. Variations of sea-level anomaly (SLA)

Fig. 2(a) shows the time series of SLA at Uchiura and Shimizu, located respectively at the eastern and western sides near the head of Suruga Bay, from 1 July to 2 September 1978. Semi-diurmal and diurnal variations are seen with low-frequency variations at both stations. The variations of SLA with semi-diurnal and diurnal periods at Uchiura are larger.

Fig. 2(a). Time series of sea level anomaly (SLA) at Uchiura (solid line) and Shimizu (broken line) from 1 July to 2 September 1978. Units are cm.
than those at Shimizu. Marked tidal variations do not exist through about two months even at Uchiura but appear weak at times from 5 August to 7 August, and from 24 August to 26 August. The maximum value between the ridge and trough of the variations for these tidal periods amounts to 18 cm at Uchiura in the latter part of July and 12 cm at Shimizu in the middle of July. The amplitudes of the semiurnal component at Uchiura are seen to be large compared with the diurnal component. Frequently, the phase of the tidal frequency at Uchiura is different from the phase at Shimizu. This indicates that the horizontal scale of SLA variations is not very large compared with the width (about 30 km) of Suruga Bay.

Fig. 2(b) shows the time series of SLA at Omaezaki and Minamiizu during the same period as for Fig. 2(a). These are the SLA records at the west and east coasts at the mouth of Suruga Bay. Though semiurnal and diurnal variations are recognized in both records, these are often weakened during the two months. The maximum value between the ridge and trough of the tidal variations is about 10 cm both at Omaezaki and Minamiizu. Fig. 2(a) and (b) shows that the tidal variations of SLA at the bay head are larger than those at the bay mouth.

The SLA variations with the tidal periods are seen to be variable throughout the two months (Fig. 2(a) and (b)). We will examine the temporal variations of SLA with these periods. So, the records are divided into several segments of 15 days each. To obtain the amplitude and phase of the four major constituents, Fourier coefficients are computed. Harmonic analysis is carried out for each 15-day record and is repeated for the
subsequent 15-day series after advancing one day at a time (Kielman and Dung, 1974; Matsuyama, 1985a).

Fig. 3 shows the temporal amplitude variations of SLA at Uchiura calculated from a 15-day series advancing one day at a time. To compare these variations with those for the temperature near the head of Uchiura Bay analyzed by the same method, the data are shown for the period from 14 July to 22 August 1978. The $M_2$ constituent is predominant throughout the whole observation period and its amplitude ranges from 4.0 to 5.3 cm. The $S_2$ constituent is placed in the second magnitude of SLA variations, but its amplitude is less than 2.2 cm. The $K_1$ constituent is the smallest of the four major constituents, while it has the second largest magnitude in sea level fluctuations (Table 1).

Table 2. Maximum and minimum values (cm) for the four major constituents of SLA at four stations from 14 July to 22 August, 1978.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Uchiura</th>
<th>Shimizu</th>
<th>Minamizui</th>
<th>Omaezaki</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td></td>
<td>5.3</td>
<td>4.0</td>
<td>2.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$S_2$</td>
<td>2.4</td>
<td>1.7</td>
<td>1.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_1$</td>
<td>2.1</td>
<td>1.5</td>
<td>0.9</td>
<td>0.3</td>
</tr>
<tr>
<td>$O_1$</td>
<td>1.2</td>
<td>0.4</td>
<td>0.9</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The maximum amplitudes of the $O_1$ and $K_1$ constituents are 1.5 cm and 0.9 cm, respectively. This indicates the predominance of semidiurnal constituents ($M_2$ and $S_2$) in SLA variations at Uchiura during July-August 1978. Table 2 shows the maximum and minimum values of temporal amplitude variations for the four major constituents of SLA at the four tidal stations during the above period, calculated by the same method as the above-mentioned harmonic analysis. The $M_2$ constituent has the largest amplitude in SLA among the four constituents both at Shimizu and Minamizui as well, but at Omaezaki the maximum amplitude of the $M_2$ constituent is comparable to or less than that of the diurnal constituents ($K_1$ and $O_1$). Amplitudes of the $M_2$ constituent range from 1.7 to 2.4 cm at Shimizu, and from 1.5 to 2.1 cm at Minamizui, which are less than the half of the amplitude at Uchiura. The $K_1$ constituent at Shimizu is the second largest and has maximum amplitude of 1.4 cm which is larger than that at Uchiura throughout the whole period. At Minamizui, the $K_1$ constituent ranges between 0.3 and 1.4 cm and the $S_2$ and $O_1$ constituents are less than 1.0 cm.

Harmonic analyses of SLA at the four tidal

Fig. 4. Temporal variations of the phase differences between RSL (observed sea level) and SLA (sea level anomaly) for the $M_2$ constituent at Uchiura (●), Shimizu (■), and Minamizui (○) calculated from a 15-day series advancing one day at a time. The estimated values refer to RSL.
stations during the summer of 1978 show that the \( M_2 \) constituent is predominant at Minamizu, Uchiura and Shimizu. So, we will focus on variations of the \( M_2 \) constituent hereafter. Fig. 4 shows the temporal variations of the phase difference between SLA and RSL for the \( M_2 \) constituent at Uchiura, Shimizu and Minamizu. The phase of SLA lags behind that of RSL by \(-30^\circ\) to \(30^\circ\) at Uchiura, by \(180^\circ\) to \(230^\circ\) at Shimizu and by \(290^\circ\) to \(310^\circ\) at Minamizu. These phase relations show that SLA is almost in phase with the raw sea level at Uchiura and out of phase with that at Shimizu. The raw sea level among the three tidal stations are nearly in phase for all four constituents shown in Table 1, so that the phase of the \( M_2 \) constituent of SLA at Uchiura is nearly out of phase with Shimizu. On the other hand, Uchiura always lags behind Minamizu by \(30^\circ\) to \(90^\circ\).

4. Comparison of the sea-level anomaly (SLA) and temperature in the subsurface layer

Long-term temperature measurements were made at Sta. O (Fig. 1) and the records were obtained at depths of 4, 12, and 16 m during the period from 14 July to 22 August 1978 (Matsuyama, 1985a). The location of Sta. O is along the coast about 1 km from the Uchiura tidal station. These temperature records were employed to obtain the temporal variations of the internal tide at Uchiura tidal station at the head of Suruga Bay. In spite of only one station, comparison between the time series of SLA and those of the subsurface temperature makes it possible to confirm whether the SLA variations at Uchiura during this period are mainly due to the internal tides or not.

Fig. 5 shows the time series of temperature at Sta. O and sea level at Sta. U from 14 July to 22 August 1978 (Matsuyama, 1985a). Temperature oscillations with the tidal periods are significant, but are not as sinusoidal as those of the sea level. The difference be-
tween maximum and minimum in the temperature variations at a depth of 16 m amounts to 6-8°C. MATSUYAMA (1985a) suggested that these temperature variations are associated with the vertical displacements of the seasonal thermocline of 35-40 m, by a rough estimation from the vertical temperature distribution.

Fig. 6 shows temporal variations of the amplitude of temperature at a depth of 16 m at Sta. O calculated by the same method as Fig. 3. This figure does not represent temporal variations of the amplitude of internal waves, but may indicate those of relative amplitudes among these constituents at each segment using the same estimation as for the vertical temperature distribution. Two notable features are found: (1) The M₂ constituent is predominant throughout the whole observational period; (2) the K₁ constituent is the smallest among the four constituents, while it has the second largest magnitude in the sea level variations (Table 1).

As might be expected, the characteristics of the temporal variations of the amplitude of temperature in the subsurface layer (Fig. 6) are similar to those of SLA at Uchiura shown in Fig. 3. Naturally both records are seen to be closely related to each other. The temperature fluctuations shown in Fig. 5 have been shown to be due to the internal tides in Uchiura Bay (MATSUYAMA, 1985a), so that those of SLA may be mainly induced by the internal tides as well.

5. Comparison of SLA and the results of numerical experiments of semidiurnal internal waves

Numerical experiments using a two-layer model have been made to study the behavior of the semidiurnal internal tides in Suruga Bay (MATSUYAMA, 1985b). The results from the numerical model give spatial variations of the amplitude and phase of the internal tides. Therefore, it is interesting to make a comparison between characteristics of SLA with the semidiurnal period at the four stations in Suruga Bay and those obtained by the numerical model. The numerical model and the results are summarized as follows. The model ocean is bounded by the coast and by an artificial boundary at the Suruga Bay mouth. The tidal fluctuations due to the internal mode were specified at the bay mouth to model the behavior of internal tides. The density difference between two layers of $3 \times 10^{-3}$ and thickness of the upper layer of 50 m are used to represent the density stratification in the summer of 1978 (MATSUYAMA, 1985 b). The amplitude of interface displacements at the mouth of Suruga Bay is obtained in the following way; the tidal current amplitude, which was obtained from current measurements in the surface layer at the moored station (Sta. P in Fig. 1) near the mouth of Suruga Bay by INABA (1982), is considered to be mainly due to internal tides, because the velocity due to the surface tides is about 0.01 cm/s at this site. The M₂ amplitude is 7.6 cm/sec from 10 July to 31 July 1978, and is 2.0 cm/sec from 31 July to 15 August 1978 (INABA, 1982). From continuity in the surface layer, amplitudes of the interfacial displacements, Z, are estimated as

$$ Z = VH/C, $$

where $V$ is the amplitude of tidal current in the upper layer, $H$ is the upper-layer thickness and $C$ is the phase velocity of internal waves. With the value in Suruga Bay in the summer of 1978, i.e. $H = 50$ m and $C = 1.2$ m/sec, $Z$ is taken as 200 cm with $V = 5.0$ cm/sec.

Fig. 7 shows the time series of interface displacement and surface elevation due to the internal tides obtained at the monitor station at Uchiura tidal station. The surface elevation of about 4 cm occurs together with the interface displacement of about 18 m and is out of phase with it. The free surface displacement is opposite phase with the thermocline displacement (PHILLIPS, 1977; LEBLOND and MYSAK, 1978).

Fig. 8 shows the co-range and co-tidal chart calculated with amplitude of 200 cm for the $M₂$ constituent. The amplitude of the interface displacements of 200 cm is taken at the bay mouth as the open boundary condition. There are three amphidromic points in Suruga Bay. The innermost one is about 7 km distant from the northern coast and about
13 km from the head of Uchiura Bay. We can see that the co-tidal lines turn cyclonically. The phases at Uchiura and Shimizu tidal stations are about 10.5 and 6.5 hours, respectively. This shows that Shimizu lags behind Uchiura by about 4 hours. The numerical experiments suggested that the internal waves with the $M_2$ constituent oscillate with nearly opposite phase between the eastern and western sides of the head of Suruga Bay under the density stratification in the summer of 1978 (Matsuyama, 1985b). This phase relation from the numerical experiment nearly agrees with that obtained from the phase difference of SLA between Uchiura and Shimizu shown in Fig. 4. The amplitude of the interface displacement under this situation is 18 m at Uchiura and 8 m at Shimizu, that is, the amplitude at Uchiura is 2.2 times as large as that at Shimizu. This ratio is consistent with the difference of SLA amplitudes between Uchiura and Shimizu for the $M_2$ constituent. Amplification of the semi-diurnal internal tides in Uchiura Bay is considered to be due to resonance to the longitudinal internal seiche in Uchiura Bay (Matsuyama, 1985b). The amplification of the $M_2$ constituent of SLA near the head of Suruga Bay agrees with that of the numerical experiment and is due to resonance with the longitudinal internal seiche of Uchiura Bay (see Fig. 8).

6. Summary and discussion

The influence of internal waves on the sea level elevation has been studied by using sea level and temperature records in subsurface layers at Uchiura located at the eastern side of the head of Suruga Bay during the period from July to August 1978. The sea level anomaly (SLA) is defined as the difference between the observed sea level (RSL) and the predicted sea level (PSL). The characteristics of SLA with the tidal periods are similar to those of the internal tides deduced from the subsurface temperature measurements at Uchiura as follows: (1) the $M_2$ constituent is
predominant, (2) the $K_1$ constituent is the smallest among the four constituents, while it is the second largest magnitude in sea level fluctuations.

The SLA variations at Uchiura, Shimizu, Omaezaki and Minamizui in Suruga Bay are analyzed during the above period. The amplitudes of SLA are large near the head with those at the mouth. The maximum amplitude of SLA for the $M_2$ constituent amounts to 5.4 cm at Uchiura, but, it is 2.4 cm at Shimizu, 2.1 cm at Minamizui, and 1.2 cm at Omaezaki. The maximum amplitudes of the other major tidal constituents ($S_2$, $K_1$, and $O_1$) of SLA range from 0.8 to 2.0 cm at all four stations. For the $M_2$ constituent, the phase at Uchiura is nearly out of phase to that at Shimizu. Numerical experiments for the $M_2$ constituent (Matsuyama, 1985b) support the amplitude difference of SLA between Uchiura and Shimizu. The phase difference of about 180 degrees (6 hours) obtained from the SLA variations between these two stations is slightly larger than that of about 120 degrees (4 hours) from the numerical experiments. These results suggest that the SLA variations with the $M_2$ constituent are mostly due to the internal tides. The amplification of the $M_2$ constituent near the head of Suruga Bay is considered to be due to resonance with the longitudinal internal seiche of Uchiura Bay (Matsuyama, 1985a).

The amplitude and phase of SLA for the $M_2$ constituent are not always in quantitative agreement with those of the internal tides deduced from temperature measurements in the subsurface layer and of the numerical experiments. The difference is considered to be mainly due to the estimations of amplitude and phase of the internal tide from temperature measurements and the numerical model. The analysis was made by assuming a prominent lowest mode of the internal tide. It is required to consider the contribution of higher modes of internal tides (Rattray et al., 1969). Long-term measurements of the vertical displacement were not made throughout the water column, but were made in the subsurface layer near the coast. Temperature measurements at only a few fixed depths in the subsurface layer (4, 12, and 16 m) cannot always give reliable information about the vertical movement of water by the internal wave because of the shallowness of the observation depths. The numerical experiments were carried out by using a two-layer model.

This study is a first step to estimate the influence of the internal wave on the sea level. Therefore, the study will be continued in the future to make long-term measurements of the vertical displacements with a thermistor chain from the surface to the bottom at the head of Suruga Bay and to obtain data with high accuracy.

References


駿河湾沿岸における海面変位に対する内部潮汐の影響

松 山 優 治・大 腦 厚

要旨：駿河湾全域の内浦湾で1978年夏に測定した表層下の水温と沿岸潮位を用いて、内部潮汐が沿岸変
動に及ぼす影響を調べた。内部潮汐による海面変位（SLA）を実測値の差として定義したところ、内部潮汐
による水温変化と SLA の主分部の特性は似ていた。 SLA の変動の最大振幅は M_2 分潮で 4.0～
5.3cm に達し、最小は K_1 分潮で 1.0cm 以下であった。次に M_2 分潮に次いで、駿河湾の内浦、清水、南
伊豆、御前崎の 4 点の潮位から、前述と同一期間の SLA を求め比較した。駿河湾近い内浦と清水は M_2 が
卓越した。振幅を比較すると、概算で内浦が清水の 2 倍、南伊豆の 2.5 倍、御前崎の 4 倍で、また内浦と清
水は逆位相の関係にあった。これは二層モデルによる M_2 に関する内部潮汐の数値実験結果と比較的良好
合うことが判った。