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A note on change of a front in an oceanic shear flow*

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Abstract: Fickian diffusion equations for concentration gradients are derived to study effects of vertical shear of the mean current and eddy diffusion processes on front-like distributions of conservative properties. The initial front is vertically uniform and is simulated with a delta function of the coordinate of mean flow which is assumed variable only in the vertical direction. In the initial stage the vertical diffusion is not important. Because of the vertical shear, the vertical gradient of the concentration is developed and increases with time. The front is advected by the mean flow but the gradient of the concentration near the front decreases with time because of diffusion processes. Horizontal gradients of salinity in the upper 50 m on the slope of the East China Sea show the Gaussian-like distribution similar to the solution of the present model. Horizontal eddy diffusivity is determined from the difference of second moments of two successive salinity gradient profiles 5 days apart, leading to $3.9\times10^2 \mathrm{m^2s^{-1}}$. A solution of the equation with both horizontal and vertical diffusivities is also determined.

1. Introduction

There are many cases in the ocean where water characteristics like temperature, salinity and nutrients change abruptly in horizontal directions. These are called fronts (BOWMAN and ESAIAS, 1978). In the upper layer the fronts are moved by the current which has usually strong vertical shear. In the bottom boundary layer the brine discharged from a diffuser at the bottom is advected by a current with a strong vertical shear and forms a front near the bottom (ICHIYE and NAKAMOTO, 1985). In both cases the concentration of the property near the front is affected both by a shearing current and diffusion processes.

This note is to show how vertical structure of a shear flow influences the vertical distribution of the concentration. The feature of the front is represented with simplest mathematical expression, a delta function. The diffusion processes are expressed by eddy diffusivities of constant values and the mean current has vertical shear only.

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2. Basic equations

The transport equation for concentration S is given by

$$\frac{\partial S}{\partial t}+\vec{u}\cdot\nabla S=K\frac{\partial^2 S}{\partial x^2}+M\frac{\partial^2 S}{\partial y^2}+N\frac{\partial^2 S}{\partial z^2}, \quad (1)$$

where $\vec{u}$ is a velocity vector with components, u, v, and w; x, and y are horizontal coordinates, ∇ is the gradient operator and K, M and N are eddy diffusivities in the x, y, and z directions.

Imposing a gradient operation on (1), we have

$$\frac{d}{dt}\nabla S=K\frac{\partial^2\nabla S}{\partial x^2}+M\frac{\partial^2\nabla S}{\partial y^2}+N\frac{\partial^2\nabla S}{\partial z^2} -(\nabla S\cdot\nabla)\vec{u}-\nabla S\times(\nabla\times\vec{u}), \quad (2)$$

where $\times$ denotes the vectorial product and $d/dt=\partial/\partial t+\vec{u}\cdot\nabla$. Equation (2) represents the vectorial equation and its component can be expressed in matrix form as

$$\begin{bmatrix} F+\partial u/\partial x & \partial v/\partial x & \partial w/\partial x \\ \partial u/\partial y & F+\partial v/\partial y & \partial w/\partial y \\ \partial u/\partial z & \partial v/\partial z & F+\partial w/\partial z \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}=0, \quad (3)$$

where

$$F=d/dt-(K\partial^2/\partial x^2+M\partial^2/\partial y^2+N\partial^2/\partial z^2), \quad (4)$$

$$X=\partial S/\partial x,\ Y=\partial S/\partial y,\ Z=\partial S/\partial z. \quad (5a, b, c)$$

3. Solutions for a front of a delta function

At the initial moment $t=0$, the front is simply represented by a delta function perpendicular to the mean current u equaling $U(z)$ which is dependent on the vertical coordinate only. Then, it is assumed that S is vertically uniform at $t=0$. Thus the initial conditions are expressed by

$$X|_{t=0}=C\delta(x),\ Z|_{t=0}=0\,. \qquad (6), (7)$$

If there is no source or sink for S, then the front is uniform in the y-direction. In the initial stage the vertical gradient is small and the vertical eddy diffusion is not effective. Then only the diffusion in the x-direction is an important dissipative process and Eq. (3) is simplified as

$$\partial X/\partial t+U(z)\partial X/\partial x=K\partial^2X/\partial x^2\,, \qquad (8)$$

$$\partial Z/\partial t+U(z)\partial Z/\partial x=K\partial^2Z/\partial x^2-U'X\,, \qquad (9)$$

where the prime denotes derivative about z.

$$X=C(2\pi Kt)^{-1/2} \times \exp\left[-\{x-U(z)t\}^2(4Kt)^{-1}\right]\,, \qquad (10)$$

$$Z=-U'tX=-CU't^{1/2}(2\pi K)^{-1/2} \times \exp\left[-\{x-U(z)t\}^2(4Kt)^{-1}\right]\,. \qquad (11)$$

Expression (11) is a particular solution of (9). Solution (10) shows that the front is advected with the velocity $U(z)$ and that its width increases but its intensity decreases with time because of the horizontal diffusion. Solution (11) indicates that the vertical gradient is developed because of the vertical shear and its magnitude at the advected front site $x=U(z)t$ increases with time, whereas the intensity of the horizontal frontal structure there diminishes with time as shown by (10).

Since X is dependent on z through $U(z)$, an effect of vertical eddy diffusivity N should be taken into account in the later stage even if the initial condition (6) indicates uniformity about z. The time when the vertical diffusivity becomes important can be estimated by comparing terms $\partial X/\partial t$ and $N\partial^2X/\partial z^2$ which is neglected in Eq. (8). Both terms become maximum at $x=U(z)t$. Therefore, the ratio of magnitudes of the two terms can be determined from their values at this point denoted with a suffix m. The ratio is given by

$$|NX''|_m/|\partial x/\partial t|_m = NK^{-1}|U'^2+UU''|t^2. \qquad (12)$$

For the values of $N/K=10^{-4}$ and $U'=10^{-2}\mathrm{s}^{-1}$ with a condition $|U'/U|\gg|U''/U'|$, the ratio (12) becomes unity for $t=10^4$ s. In case of the uniform shear $U'=a=$const., an analytical solution of X can be obtained for Eq. (8) with the vertical eddy diffusion term as an integral form instead of (10) (Appendix). A particular solution of (9) with the same vertical eddy diffusion term is given by the second term of (10) with the constant shear a.

4. Application

An actual front cannot be represented by a delta function even in its generating period. Its movement can not be expressed by a simple advection as modeled here either. However, some features of front-like distributions of properties can be explained by the present model. For instance, MOOERS et al. (1978) classified topology of prograde and retrograde fronts. This may be explained as effects of the shear in the mean flow. When the concentration gradients are expressed by (10) and (11), the slope of the concentration isolines dx/dz can be given by

$$dx/dz=-Z/X=U't\,. \qquad (13)$$

This relation is valid even when the vertical diffusion is considered as in Appendix. Relation (13) simply indicates that the isolines are advected with the mean current $U(z)$ even with the dissipative processes, though the gradient of the concentration normal to the isolines decreases with time according to (10) and (11). Therefore, the six basic patterns of the retrograde and prograde fronts classified by MOOERS et al. (1978) can be attributed to different vertical distributions of shear in the mean flow normal to the fronts.

In order to apply the solution given by Eq. (10), vertical distributions of salinity on a continental slope will be used, because temperature and nutrients are influenced by other factors like heating, cooling, consumption, etc. than advection and diffusion. These distributions must be determined within a relatively short

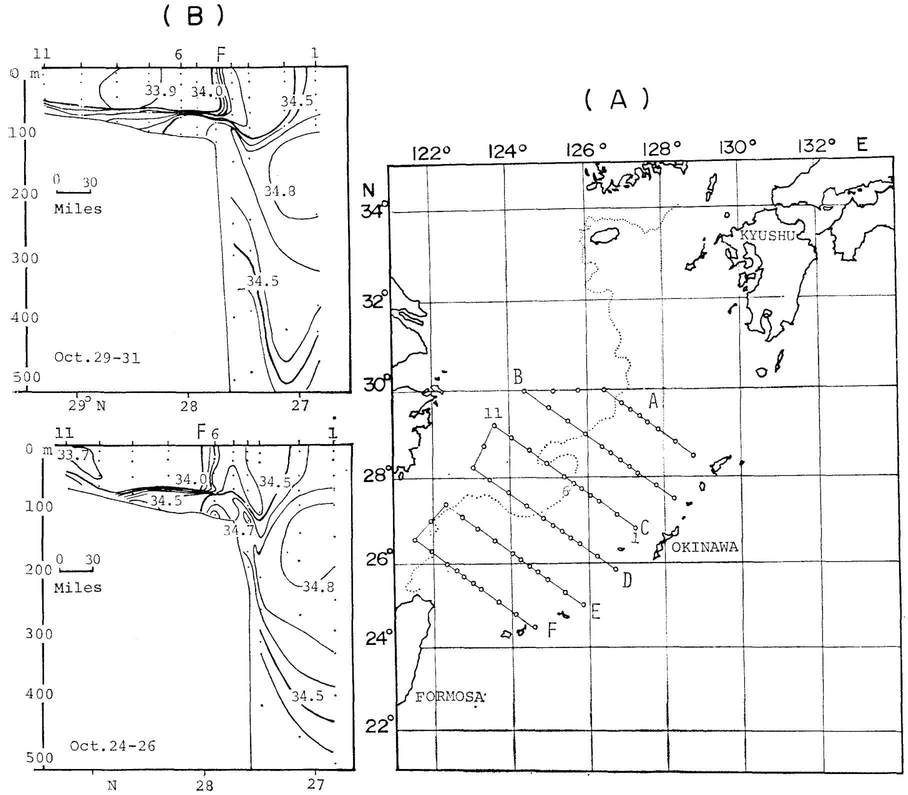

Fig. 1. (A) Salinity sections of C Line in October 1973 measurements (modified from originals). Left: October 24–26, (SI) right: October 29–31 (SII). F indicates front. (B) Location of C Line.

period of time in order to eliminate other factors like the lateral advection. Figure 1 indicates two succesive vertical salinity sections on October 24 to 26 and 29 to 31 in 1973 along a line over the continental slope in the East China Sea and their location (Japan Meteorological Agency, 1976). The two sections are denoted as SI and SII and the isohalines are modified from the original ones in order to clarify the frontal structures.

The gradient $X=\partial S/\partial x$ is almost uniform vertically in the upper 50 m. Its values for SI and SII are plotted in Fig. 2 with the origin for SII being offset by 52 km northwestward. The two profiles of the gradient show Gaussian-like distributions about x. The profile for SI has higher peak and narrower width, whereas the one for SII is flattened, suggesting the diffusion process as indicated by Eq. (10).

First, the advective velocity can be determined

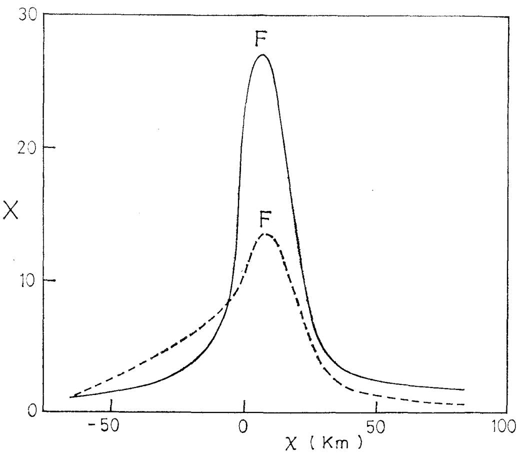

Fig. 2. Salinity gradient $X=\partial S/\partial x$ against the distance x along C Line (km). Full line for Oct. 24–26 (SI), dashed line for Oct. 29–31 (SII). F indicates front.

from the northwestward displacement of the peak gradient, which is 51.9 km in about 5 days. This yields the mean velocity in the upper 50 m as 12.0 cm s^{-1} which is comparable with the mean velocity of 0.23 kts in the direction of the line determined from the average of GEK data of SI and SII (Japan Meteorological Agency, 1976).

When X is expressed as an arbitrary function of a distance $s(=x)$ as shown in Fig. 2, the second moment can be obtained as

$$m_2=(\bar{X})^{-1}\int X(s-m_1)^2\,ds\,, \qquad (14)$$

with

$$\bar{X}=L^{-1}\int X ds\,,\quad m_1=(\bar{X})^{-1}\int Xs\,ds. \qquad (15), (16)$$

In these integrals the limits of integration are truncated at 5 % of the peak value of X and L denotes the integration interval thus defined.

The following table lists $\bar{X}$, m_1 $\overline{s^2}$ and m_2 for SI and SII $(\overline{s^2}=(\bar{X})^{-1}\int Xs^2\,ds)$.

Table 1.

	$\bar{X}$(‰m^{-1} $\times 10^{-6}$)	m_1(m $\times 10^3$)	$\overline{s^2}$(m^2 $\times 10^6$)	m_2(m^2 $\times 10^6$)
SI	10.5	5.0	287	262
SII	5.6	1.3	432	431

If X is given by (10) and integration limits of (14) are taken to infinity, then

$$m_2=Kt\,. \qquad (17)$$

Actual integration of (14) to (16) has to be truncated for profiles as shown in Fig. 2 and a correction for the truncation is needed to (17) as discussed by ICHIYE and NAKAMOTO (1985). Since observed values of X as a function of s are only approximate, such a correction may be superfluous and ambiguous.

The horizontal eddy diffusivity K can be determined from

$$m_2(II)-m_2(I)=K(t_{II}-t_I)\,, \qquad (18)$$

where I and II represent SI and SII, respectively. The values of M from Table 1 yield $K=3.9\times 10^2$ m^2s^{-1} which is reasonable.

Salinity section of SII of Fig. 2 indicates much sharpened halocline at about 60 m on the shelf. This seems to prove the increase of Z with time as expressed by (11) or change of isohaline slope as shown by (13). However, there are no sufficient data about the current near the bottom, thus further discussion is refrained. We hope more satisfactory data will be obtained in the future.

If the mean velocity is assumed to decrease to zero linearly with depth from 60 m to the bottom, Eq. (13) yields $dx/dz \sim 1.3\times 10^3$ or the isohalines will be almost horizontal as the data show.

5. Conclusion

More applications of the present model will become possible when more data becomes available about vertical structures of the mean current and details of vertical profiles of salinity and other conservative properties together with their changes with time. The present model does not explain the distributions near the bottom which have different characteristics both in the mean current and diffusion processes. We will treat the latter problem in the future.

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Appendix

A solution of (8) with the vertical diffusion term $N\partial^2X/\partial z^2$ and for the initial condition (6) and with the uniform vertical shear $U'=\alpha$ is given by

$$\begin{aligned} X(x,z,t) &= C\int_{-h}^{0}(KN)^{-1/2}(4\pi t)^{-1}(1+NK^{-1}\alpha^2t^2/12)^{-1/2} \\ &\cdot \exp\left[-\frac{\{x-U_0t-1/2\alpha t(Z-Z_0)\}^2}{4Kt(1+NK^{-1}\alpha^2t^2/12)} -\frac{(Z-Z_0)^2}{4Nt}\right]dz_0, \end{aligned} \qquad (A\cdot 1)$$

where $U=U_0+\alpha z$ and subscripts 0 and $-h$ indicate the surface and the bottom, respectively. The integrand of (A·1) is a solution for the initial condition

$$X|_{t=0}=C\delta(x)\delta(z-z_0). \qquad (A\cdot 2)$$

The integral of (A·1) can be expressed with error functions erf and erfc (ABRAMOWITZ and STEGUN, 1964) but further discussion will be given elsewhere.

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シャーのある流れの中のフロントの変化についての小論

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要旨: 保存量の不連続的な分布に及ぼす平均流の鉛直シャーを渦拡散の影響を見るため濃度の傾度のフイック形の拡散方程式を導いた。初期のフロントは鉛直には均質で平均流の方向の座標のデルタ関数で表わした。平均流は鉛直方向にだけ変るとした。初めの間は鉛直拡散は重要でない。濃度に鉛直傾度が生じ，時間と共に増える。フロントは平均流で流されるが拡散のため濃度の傾度は時間と共に減少する。東支那海の上層 50 米の塩分の水平傾度は拡散方程式の解のようにガウス分布を示している。5日離れた2つの塩分傾度の分布から求めた水平拡散係数は $3.9\times10^2\,\mathrm{m^2\,s^{-1}}$ であった。鉛直方向の拡散係数も入った方程式の解も求めることができた。