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Parameterization of the eddy diffusivity due to double diffusive convection

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Abstract : The turbulent energy dissipation rates ε in the western North Pacific Ocean were observed using a microstructure profiler at 49 casts, and the measured values were converted into diffusivities of heat K_T and salt K_S. We obtained a new relationship between the Richardson number R_i and the buoyancy Reynolds number R_{eb}, which enables us to use R_i, instead of R_{eb}, as an indicator for distinguishing double diffusive convection from turbulence. We further obtained new relationships between K_S, K_T, R_i and the density ratio R_ρ by improving the parameterization proposed by KIMURA *et al.* (2011).

Keywords : *eddy diffusivity, double diffusive convection, Richardson number*

1. Background

How can we estimate the eddy diffusivity from general hydrographic observation data? If we can estimate the eddy diffusivity, the knowledge contributes to elucidate modification processes of water masses and to improve large–scale general ocean circulation models (*e.g.* BRYAN 1987; GARGETT and HOLLOWAY 1992).

GARGETT and HOLLOWAY (1992) used different diffusivities for heat and salt in GFDL ocean general circulation model. This difference in diffusivities between heat and salt is produced by double diffusive convection. Their results showed a formation of salinity minimum in the upper ocean. TALLEY and YUN (2001) investigated modification processes of water mass in the perturbed region between the Kuroshio and Oyashio. They showed that double diffusive convection and cabbeling increase its density. These processes should have effects on producing water mass having salinity minimum called NPIW (North Pacific Intermediate Water).

In the upper ocean, however, turbulence and double diffusive convection can co-exist. As a result, it is arduous to distinguish the role of double diffusive convection from that of turbulence. Thus, INOUE *et al.* (2007) discussed this point in detail. They conducted microstructure observations focusing on eddy diffusivities of salt K_S and heat K_T in the perturbed regions where turbulence and double diffusive convection both contribute to mixing. They proposed a simple eddy diffusivity model to account properly for activity of turbulence and double diffusive convection. They also used the combination of the buoyancy Reynolds number R_{eb} and the density ratio R_ρ which enables us to distinguish double diffusive convection from turbulence. When R_{eb} is below 20 and R_ρ is between 0.5 and 2.0, they suggested that double diffusive

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convection should prevail.

R_{eb}, however, must be calculated from the energy dissipation rate obtained by direct micro-structure measurements. Therefore, R_{eb} cannot be used commonly in the parameterization of eddy diffusivity and many researchers considered the effect of double diffusive convection by using R_ρ only. For example, TOYAMA and SUGA (2012) found that salt finger or turbulence contributed to the formation and maintenance of central mode water. They only used R_ρ to discuss the effects of double diffusive convection. From the point of view of micro-scale mixing studies, their study was not completed because the density ratio indicates activity of double diffusive convection and can not discriminate that from turbulence. The effect of double diffusive convection was not evaluated clearly. Inaccuracy of the mechanism for the formulation and maintenance of central mode water remains to be solved. If we have other indicators which can distinguish double diffusive convection from turbulence, such indicators enable researchers to evaluate the effect of micro-scale mixing precisely. Thus, we need other parameters calculated from general hydrographic measurements to distinguish double diffusive convection from turbulence.

In this context, following LOZOVATSKY and FERNANDO (2012) which discussed the relationship between the Richardson number R_i and R_{eb} in the atmosphere in the Salt Lake City, we evaluate R_i whether it could be used instead of R_{eb} in the ocean (when R_i is below 0.25, turbulence occurs (e.g., THORPE 2005)).

R_i and R_ρ are also used in parameterizations of eddy diffusivity. For example, KIMURA *et al.* (2011) proposed eddy diffusivity parameterizations with R_i and R_ρ by the direct numerical simulation. They directly simulated salt finger convection in fine grids by changing R_ρ and R_i. They considered some cases whether the field is statically stable or not. They obtained relationships among K_S, K_T, R_i and R_ρ; however the parameterizations were not evaluated by observation data.

Consequently, we focus on the Richardson number R_i calculated by CTD and LADCP (Lowered Acoustic Doppler Current Profiler) data which are commonly used in hydrographic observations. Then, we discuss the relationship between R_i and R_{eb}. If we can use R_i instead of R_{eb}, it becomes easier to distinguish double diffusive convection from turbulence.

2. Analysis method

Our observations were conducted in the R/V Hakuho-maru of the Japan Agency for Marine-Earth Science and Technology (JAMSTEC) during three periods, namely, Nov. 2005 (KH05-4 cruise), May 2007 (KH07-1 cruise) and Oct. 2008 (KH08-3 cruise) (Fig.1). We obtained energy dissipation rates ε to estimate eddy diffusivities using a microstructure profiler called TurboMAP (Turbulence Ocean Microstructure Acquisition Profiler). It has two shear probes, and one fast response thermistor, and 49 casts were conducted. TurboMAP was dropped freely down to a depth of about 600 db. CTD (SBE) and LADCP observations were also conducted simultaneously at each TurboMAP station.

2.1. Identification of double diffusive convection

We calculated density ratio R_ρ and Turner angle T_u. First, R_ρ is defined as

$$R_\rho = \alpha \frac{\partial \theta}{\partial z} \Big/ \beta \frac{\partial S}{\partial z}, \tag{2.1}$$

where α is the thermal expansion and β is the haline contraction coefficients, respectively. $\partial\theta/\partial z$ and $\partial S/\partial z$ are the mean vertical gradients of potential temperature and salinity, respectively. Then, T_u is defined by R_ρ as

$$T_u = \tan^{-1} \frac{R_\rho + 1}{R_\rho - 1}. \tag{2.2}$$

Salt finger convection is active when $1 < R_\rho < 2$ ($72^0 < Tu < 90^0$). Diffusive convection is also active when $0.5 < R_\rho < 1$ ($-90^0 < Tu < -72^0$). KANTHA *et al.* (personal communication) proposed to use 'Circle diagram' together with Tu. Using this diagram, we can easily judge whether double diffusive convection is active or not. We will use this diagram in the

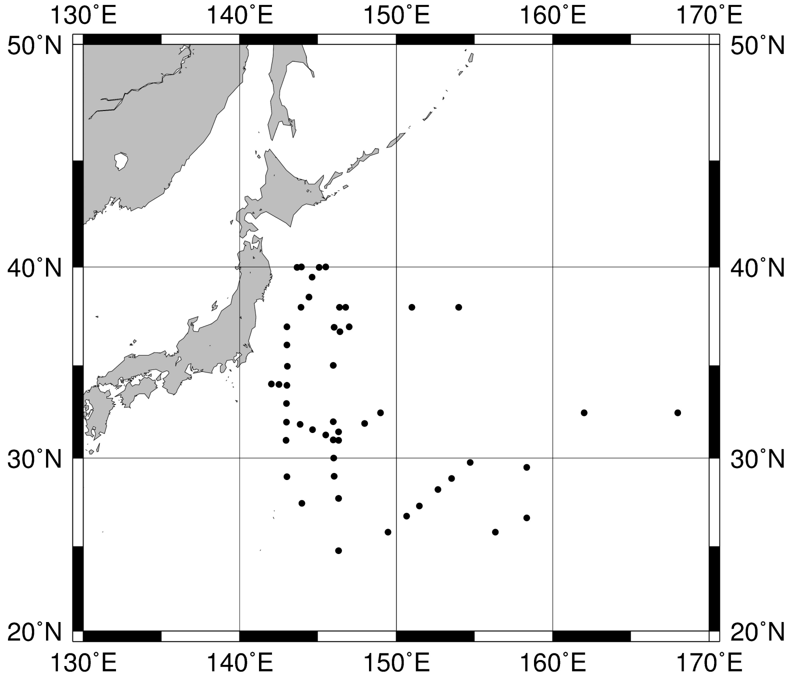

Fig. 1. A map of stations.

following section.

2.2. Energy dissipation rate

We obtained ε, following the relation obtained by OSBORN (1980),

$$\varepsilon = \frac{15}{2}\nu\overline{\left(\frac{\partial u'}{\partial z}\right)^2}, \tag{2.3}$$

where ν is the molecular viscosity ($\sim 10^{-6}$ m^2/s), $\partial u'/\partial z$ is the vertical shear of the horizontal velocity fluctuations with the over bar denoting the ensemble average.

2.3. Eddy diffusivities

We estimated eddy diffusivities when double diffusive convection was active. Following INOUE *et al.* (2007), in order to distinguish double diffusive convection from turbulence, we used the buoyancy Reynolds number R_{eb} defined by

$$R_{eb} = \frac{\varepsilon}{\nu N^2}, \tag{2.4}$$

where N is the buoyancy frequency. When R_{eb} is below 20, double diffusive convection is effective to enhance mixing (e.g., PADMAN and DILLON 1987; GREGG, 1988; INOUE *et al.* 2007; KANTHA *et al.* (personal communication)). In the present study, when R_ρ is between 1 and 2, and R_{eb} is below 20, K_S^{SF} and K_T^{SF} are estimated by

$$K_S^{SF} = \left(\frac{R_\rho - 1}{1-\gamma^{SF}}\right)\frac{\varepsilon}{N^2}, \tag{2.5}$$

$$K_T^{SF} = \left(\frac{\gamma^{SF}}{R_\rho}\right)K_S^{SF}, \tag{2.6}$$

(*e.g.* KELLEY 1986); when R_ρ is between 0.5 and 1, and R_{eb} is below 20, and K_S^{DC} and K_T^{DC} are estimated by,

$$K_S^{DC} = \gamma^{DC}R_\rho K_T^{DC} = \frac{\gamma^{DC}(1-R_\rho)}{1-\gamma^{DC}}\frac{\varepsilon}{N^2}, \tag{2.7}$$

$$K_T^{DC} = \frac{1}{1-\gamma^{DC}}\frac{1-R_\rho}{R_\rho}\frac{\varepsilon}{N^2}, \tag{2.8}$$

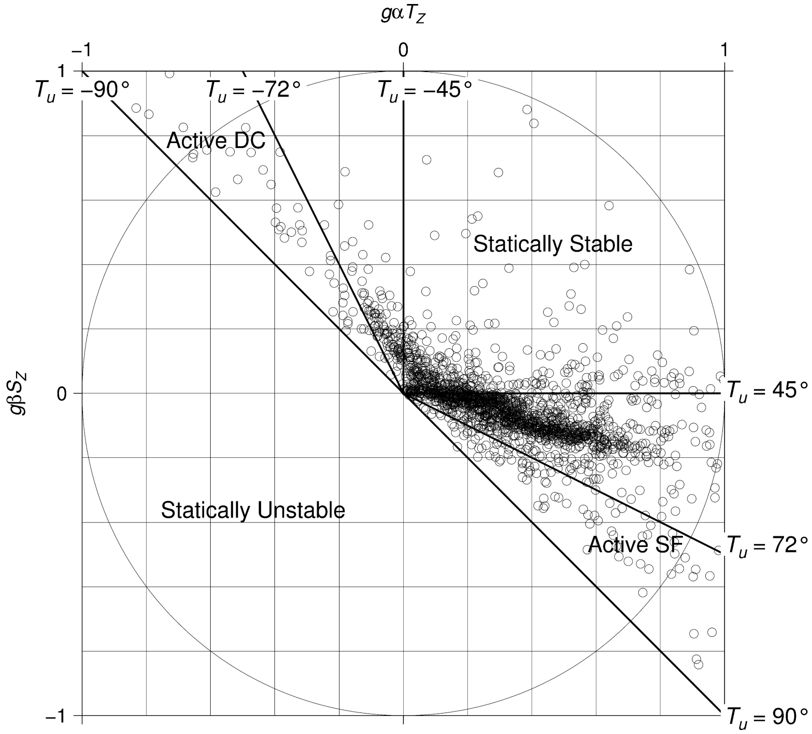

Fig. 2. Circle diagram. Vertical salinity gradient ($g\beta S_z$) and temperature gradient ($g\alpha\theta_z$) are taken in horizontal and vertical axis, respectively. Solid lines show the value of Turner angle. Small circles are observed layers.

(e.g. KELLEY 1984), where *SF* stands for Salt Finger convection, *DC* stands for Diffusive Convection, γ is the density flux ratio due to double diffusive convection defined by

$$\gamma = \frac{\alpha F_T}{\beta F_S}, \tag{2.9}$$

with αF_T and βF_S the vertical density fluxes due to heat and salt, respectively, and related to R_ρ such that

$$\gamma^{SF} = \sqrt{R_\rho}\left(\sqrt{R_\rho} - \sqrt{R_\rho - 1}\right), \text{ KUNZE (1987)}, \tag{2.10}$$

$$\gamma^{DC} = \frac{1/R_\rho + 1.4(1/R_\rho - 1)^{3/2}}{1 + 14(1/R_\rho - 1)^{3/2}}, \text{ KELLEY (1990)}. \tag{2.11}$$

By the definition (OSBORN 1980), we obtained eddy diffusivities due to turbulence as

$$K_\rho^{Turb} = K_T^{Turb} = K_S^{Turb} = \frac{Rf}{1-Rf}\cdot\frac{\varepsilon}{N^2} = \Gamma\cdot\frac{\varepsilon}{N^2}. \tag{2.12}$$

Here, Rf is the flux Richardson number assumed to be 0.17, then the mixing efficiency Γ becomes 0.2 for isotropic turbulence (SCHMITT *et al.* 2005). *Turb* stands for Turbulence. Hereafter, we use K_T^{Obs} as representation of K_T^{Turb}, K_T^{SF} or K_T^{DC}. K_S^{Obs} also of K_S^{Turb}, K_S^{SF} or K_S^{DC}. *Obs* stands for the observation value.

2.4. The Richardson number

The Richardson number R_i was calculated using the buoyancy frequency N and the vertical shear of horizontal velocity S, both defined at 10 m vertical scale, such that

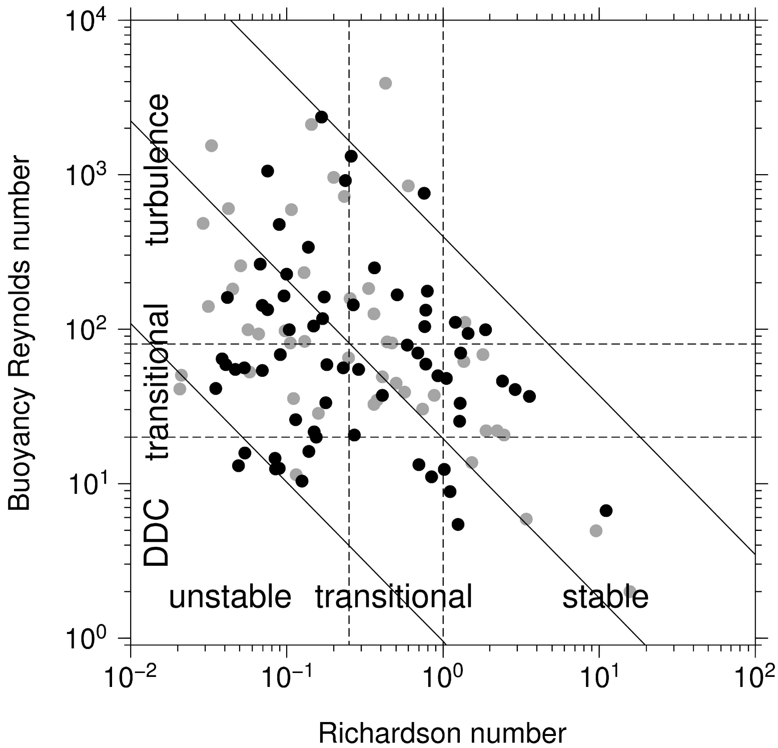

Fig. 3. Relationship between R_{eb} and R_i when double diffusive convection occur ($0.5 \leqq R_\rho < 1$: gray, $1 \leqq R_\rho < 2$: black). Middle solid line shows EOF first mode. Contribution of EOF first mode is 72%, and that of EOF second mode is 28%. The upper and lower limits indicate 95% confidence interval.

$$R_i = \frac{N^2}{S^2} = \frac{N^2}{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}, \tag{2.13}$$

where u and v are horizontal velocities, respectively.

3. Results and discussion

3.1. Activity of double diffusive convection

The circle diagram plot shows that double diffusive convection was not so active in our observation area (Fig. 2). The percentage of active double diffusive convection layer was about 10%. A large amount of data clustered in the weak SF ($2 < R_\rho$) and DC ($R_\rho < 0.5$) areas.

3.2. Comparison of the Richardson number and the buoyancy Reynolds number

When double diffusive convection is active ($0.5 < R_\rho < 2$), the relationship between R_{eb} and R_i (Fig. 3) is

$$R_{eb} = 19.5 R_i^{-1.03}. \tag{3.1}$$

When R_{eb} is about 20, R_i is unity. This means that this value of R_{eb} indicates the possibility of the layer becomes whether stable or not. However, when we consider the flux Richardson number as a criterion of turbulence through the energy argument, the criterion of R_i should be 0.25. When we put this value into eq. (3.1), R_{eb} is about 80. By the histograms of R_i and R_{eb}, modes of R_i and R_{eb} take these values (Fig. 4).

Fig. 3 also indicates that even when the range of R_{eb} is between 20 and 10^3, double diffusive convection should occur. In this range the turbulence should suppress the onset of double diffusive convection; however, Taylor

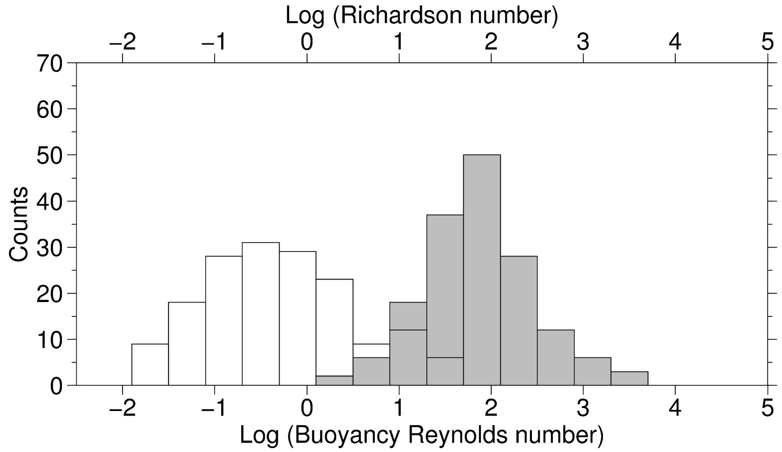

Fig. 4. Histograms of the Richardson number (white) and the buoyancy Reynolds number (gray).

(1991) showed that turbulence and double diffusive convection might co-exist under the same situation since salt finger convection appears rapidly after it is destroyed by turbulence. SMYTH *et al.* (2005) also indicated that values of K_S and K_T are different when R_{eb} is less than O (10^2). Thus, when R_i is over 0.25 (R_{eb} is under 80), K_S and K_T should be affected by double diffusive convection. In the next section, we use R_i instead of R_{eb}.

3.3. Parameterization proposed by Kimura *et al.* (2011)

We compared K_S^{Obs} and K_T^{Obs} calculated in this study with those obtained by KIMURA *et al.* (2011) parameterizing eddy diffusivities by DNS with R_i and R_ρ (hereafter, we call this as DNS parameterization). DNS parameterizations are conducted in limited situations; therefore it cannot be applied to observational results. However, we adapted their functional form to our observation and tried to compare with microstructure data.

When $1<R_\rho<2$, DNS parameterizations for K_S and K_T are expressed as

$$K_S^{SF,DNS}(R_\rho, R_i) = 4.38\times10^{-5}R_\rho^{-2.7}R_i^{0.17}, \quad (3.2a)$$

$$K_T^{SF,DNS}(R_\rho, R_i) = 3.07\times10^{-5}R_\rho^{-4.0}R_i^{0.17}. \quad (3.2b)$$

When R_ρ becomes large, $K_S^{SF,DNS}$ and $K_T^{SF,DNS}$ become small. When R_i becomes large, $K_S^{SF,DNS}$ and $K_T^{SF,DNS}$ become large. When we put observed R_i and R_ρ into DNS equations (Fig. 5a, b), $K_S^{SF,DNS}$ and $K_T^{SF,DNS}$ (small black circles) are found to be smaller than K_S^{Obs} and K_T^{Obs} (large black squares with error bars). Particularly, if we applied DNS parameterization when R_ρ is under 5, $K_T^{SF,DNS}$ is obviously underestimated because it becomes small rapidly due to the functional dependence of R_ρ. However, dependences on R_ρ of $K_S^{SF,DNS}$ and $K_T^{SF,DNS}$ are similar to K_S^{Obs} and K_T^{Obs} (Fig. 5a, b).

Here, the average value of the eddy diffusivity in the upper 1000 m is about $(2-4)\times10^{-5}$ m^2/s (WATERHOUSE *et al.* 2014). If we use ordinary functional form of $K_T^{SF,DNS}$, it becomes lower than the average value. This means that DNS parameterizations are not applied to oceanic data directly. Thus, we changed the functional form of $K_T^{SF,DNS}$ to the same form as that of $K_S^{SF,DNS}$ because the functional form of $K_S^{SF,DNS}$ was good performer when we use DNS parameterization with R_ρ under 5 (Fig. 5a, b).

Then we calculated coefficients in order to fit to the observed values by following equations.

$$K_S^{Obs} = C^S R_\rho^{-2.7}R_i^{0.17}, \quad (3.3a)$$

$$K_T^{Obs} = C^T R_\rho^{-2.7}R_i^{0.17}, \quad (3.3b)$$

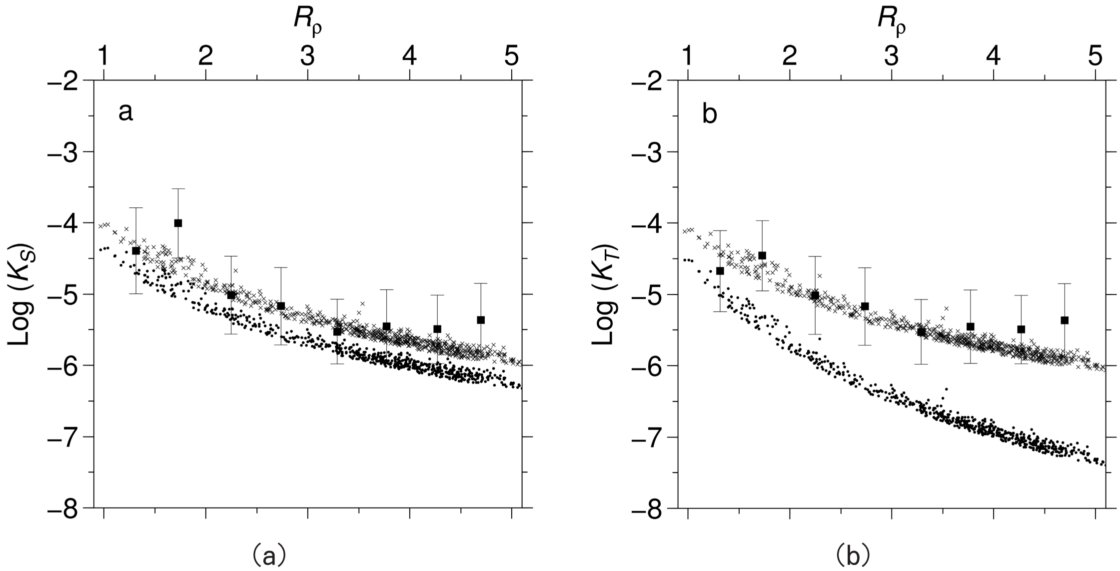

Fig. 5. Relationships between K_T, K_S and R_ρ. Squares show the average of observation results, and vertical lines show error bars. Black circles show the original DNS results. Black crosses show the improved DNS results. (a: K_S, b: K_T).

where C^S and C^T are the coefficients of each layers, and

$$K_S^{Obs} = AR_\rho^{-2.7}R_i^{0.17}, \tag{3.4a}$$

$$K_T^{Obs} = BR_\rho^{-2.7}R_i^{0.17}. \tag{3.4b}$$

Here,

$$A = \frac{\sum_{i=1}^{n} C_i^S}{n}, \tag{3.5a}$$

$$B = \frac{\sum_{i=1}^{n} C_i^T}{n}, \tag{3.5b}$$

where n is a number of layers. Then, we can finally obtain the new relationships

$$K_S^{Imp}(R_\rho^{Obs}, R_i^{Obs}) = 9.35\times10^{-5}R_\rho^{-2.7}R_i^{0.17}, \tag{3.6a}$$

$$K_T^{Imp}(R_\rho^{Obs}, R_i^{Obs}) = 7.61\times10^{-5}R_\rho^{-2.7}R_i^{0.17}. \tag{3.6b}$$

We can confirm that improved DNS parameterizations agree fairly well with the observed results (Fig. 5a, small black crosses).

5. Conclusion

We have estimated and parameterized the eddy diffusivity. As a result, we have obtained a new relationship between the Richardson number R_i and the buoyancy Reynolds number R_{eb}, which enables us to use R_i as an indicator distinguishing double diffusive convection from turbulence instead of R_{eb}.

Likewise, new relationships of K_T or K_S focusing R_ρ and R_i have been determined by improving the DNS parameterization proposed by Kimura *et al.* (2011). Thus, we can estimate the effect of salt finger convection by the fine scale parameter.

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References

Bryan, F. O. (1987): Parameter sensitivity of primitive equation ocean general circulation models. J. Phys. Oceanogr., **17**: 970–985.

Gargett, A. E. and G. Holloway (1992): Sensitivity of the GFD model to different diffusivities for heat and salt. J. Phys. Oceanogr., **22**: 1158–1177.

Gregg, M. C. (1988): Mixing in the thermohaline staircase east of Barbados, in Small- Scale Turbulence and Mixing in the Ocean, J. C. J. Nihoul and B. M. Jamart, Eds., Elsevier Oceanography Series, Vol. **46**. pp. 453–470.

Inoue, R., H. Yamazaki, F. Wolk, T. Kohno and J. Yoshida (2007): An estimation of the buoyancy flux for a mixture of turbulence and double diffusion. J. Phys. Oceanogr., **37**: 611–624.

Kantha, L, S. Carniel, M. Sclavo and A. Bergamasco (2013): On identifying double diffusive mixing in the ocean. (Unpublished manuscript)

Kelley, D. E. (1984): Effective diffusivities within oceanic thermocline staircases. J. Geophys. Res., **89**: 10484–10488.

Kelley, D. E. (1986): Oceanic thermocline staircases. PhD Dissertation, Dalhousie University. 333 pp.

Kelley, D. E. (1990): Fluxes through diffusive staircases: a new formulation. J. Geophys. Res., **95**: 3365–3371.

Kimura, S., W. D. Smyth and E. Kunze (2011): Turbulence in a sheared, salt-fingering-favorable environment: anisotropy and effective diffusivities. J. Phys. Oceanogr., **41**: 1144–1159.

Kunze, E. (1987): Limits on growing finite-length salt fingers: a Richardson number constraint. J. Mar. Res., **45**: 533–556.

Lozovatsky, I. D. and H. J. S. Fernando (2012): Mixing efficiency in natural flow. Phil. Trans. R. Soc. A, 2013 371, doi:10.1098/rsta.2012.0213.

Osborn, T. R. (1980): Estimates of the local-rate of vertical diffusion from dissipation measurements. J. Phys. Oceanogr., **10**: 83–89.

Padman, L. and T. M. Dillon (1987): Vertical heat fluxes through the Beaufort Sea thermohaline staircase. J. Geophys. Res., **92** : 10799–10806.

Schmitt, R. W., J. R. Ledwell, E. T. Montgomery, K. L. Polzin and J. M. Toole (2005): Enhanced diapycnal mixing by salt fingers in the thermocline of the Tropical Atlantic. Science **308**: 685–688.

Smyth, W. D., J. D. Nash and J. N. Moum (2005): Differential diffusion in breaking Kelvin-Helmholtz billows. J. Phys. Oceanogr., **35**: 1004–1022.

Talley, L. D. and J. Y. Yun (2001): The role of cabbeling and double diffusion in setting the density of the North Pacific intermediate water salinity minimum. J. Phys. Oceanogr., **31**: 1538–1549.

Taylor, J. (1991): Laboratory experiments on the formation of salt fingers after the decay of turbulence. J. Geophys. Res., **96**: 12497–12510.

Thorpe, S. A. (2005): The turbulent ocean. Cambridge University Press, 439 pp.

Toyama, K. and T. Suga (2012): Roles of mode waters in the formation and maintenance of central water in the North Pacific. J. Oceanogr., **68**: 79–92.

Waterhouse, A., J. MacKinnon, J. Nash, M. Alford, E. Kunze, H. Simmons, K. Polzin, L. St. Laurent, O. Sun, R. Pinkel, L. D. Talley, C. Whalen, T. Huussen, G. Cater, I. Fer, S. Waterman, A. Naveira-Garabato, T. Sanford and C. Lee (2014): Global patterns of diapycnal mixing from measurements of the turbulent dissipation rate. J. Phys. Oceanogr., doi:10.1175/JPO-D-13-0104.1 (in press).

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